sample quiz #5

1. $\begin{vmatrix} 1 & x & x^2 \\ x & 1 & x^2 \\ x^2 & x & 1 \end{vmatrix} = 1-x^3 - x(x-x^3) + x^2(x^3-x^2)$

$= 1 - x^3 - x^2 + x^4 + x^5 - x^4$

$= x^5 - x^3 - x^2 + 1$

2. $\mathbb{K}[x]$ of deg n is not a vector space because it doesn't satisfy the addition axiom

$x^n + -x^n = 0$ which is not in the set of polynomials w/ deg n

$0x^n + 0x^{n-1} \dots$ is the 0 vector, which is contained in the set of polynomials w/ deg $\leq n$

for any polynomial, $p(x) + 0 = p(x)$

any 2 polynomials added together are part of the set of $p(x)$ w/ deg $\leq n$

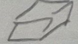
$\underbrace{ax^n + bx^{n-1} + \dots zx + z_0}_{p(x)} + \underbrace{\alpha x^n + \beta x^{n-1} \dots}_{p'(x)} = Cx^n +$ different constant $x^{n-1} \dots$ which is a poly of deg $\leq n$

any polynomial of deg n multiplied by a scalar multiple

$ax^n \Rightarrow bx^n \in p(x)$ of deg $\leq n$

therefore, polynomials of deg $\leq n$ form a subspace

3. find $T: \mathbb{R}^3 \to \mathbb{R}^3$ st the vol of parallelepiped defined by $T(1,0,0), T(0,1,0), T(0,0,1)$ is $1/2$

vol of  is det(A), with A being the vectors of the parallelepiped

so scaling the volumes to $1/2$ means the determinant of the new parallelepiped by $1/2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{vmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1/2$$

$$\boxed{\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$