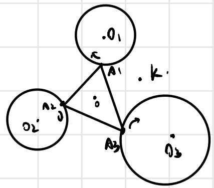
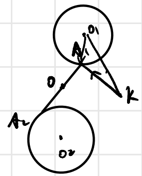
HW 1

6.



$\Leftarrow$

simplified segment case



$\vec{KO} = \frac{1}{3}(\vec{KA_1} + \vec{KA_2} + \vec{KA_3})$

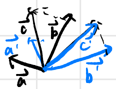
$= \underbrace{\frac{1}{3}(\vec{KO_1} + \vec{KO_2} + \vec{KO_3})}_{\vec{u}} + \underbrace{\frac{1}{3}(\vec{O_1A_1} + \vec{O_2A_2} + \vec{O_3A_3})}_{\vec{v}}$

for abbreviation, let's see $\vec{u}$.

$\vec{u}$ is clearly a vector w/ fixed length & direction

for $\vec{v}$:

$\because$ for two vectors $\vec{a}$ and $\vec{b}$ with fixed lengths and same angular velocity



geographically, their sum $\vec{c}$ should be a vector with also fixed length and angular velocity

$\therefore$ the addition can be generalized from 2 to n

$\therefore$ $\vec{v}$ is a vector with fixed length and constant angular velocity

same with the segment case

it forms a circle

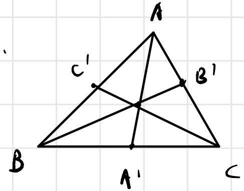
$\vec{KO} = \frac{1}{2}(\vec{KA_1} + \vec{KA_2})$

$= \frac{1}{2}(\vec{KO_1} + \vec{O_1A} + \vec{KO_2} + \vec{O_2A})$

$= \underbrace{\frac{1}{2}(\vec{KO_1} + \vec{KO_2})}_{\text{a vector w/ fixed length \& direction}} + \underbrace{\frac{1}{2}(\vec{O_1A} + \vec{O_2A})}_{\text{a vector w/ fixed length \& direction changing at } w}$

= circle shape movement with angular velocity w

8.



$\because$ Based on Q7: $\overrightarrow{AA'} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{AC})$

$\therefore \overrightarrow{AA'} + \overrightarrow{BB'} + \overrightarrow{CC'} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{BC} + \overrightarrow{BA} + \overrightarrow{CA} + \overrightarrow{CB})$

$= 0$

$\therefore$ a triangle can be formed

11. $A(x_a, y_a)$ $B(x_b, y_b)$, C; D E.

$\overrightarrow{AB} = (x_b - x_a,\ y_b - y_a)$ $\quad M(\frac{x_a + x_b}{2}, \frac{y_a + y_b}{2})$

$\overrightarrow{CD} = (x_d - x_c,\ y_d - y_c)$ $\quad N(\frac{x_c + x_d}{2}, \frac{y_c + y_d}{2})$ $\quad Q(\frac{x_a + x_b + x_c + x_d}{4}, \frac{\cdots}{4})$

$\overrightarrow{BC} = (x_c - x_b,\ y_c - y_b)$ $\quad O(\frac{x_b + x_c}{2}, \frac{y_b + y_c}{2})$

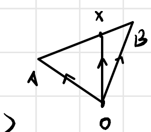
$\overrightarrow{DE} = (x_e - x_d,\ y_e - y_d)$ $\quad P(\frac{x_d + x_e}{2}, \frac{y_d + y_e}{2})$ $\quad R(\frac{x_b + x_c + x_d + x_e}{4}, \frac{\cdots}{4})$

$\overrightarrow{AE} = (x_e - x_a,\ y_e - y_a)$

$\overrightarrow{QR} = (\frac{x_e - x_a}{4}, \frac{y_e - y_a}{4}) = \frac{1}{4}\overrightarrow{AE}$

$\therefore$ parallel

12. ① X lie on AB $\Rightarrow \overrightarrow{OX} = \lambda\overrightarrow{OA} + (1-\lambda)\overrightarrow{OB}$



suppose $\overrightarrow{AX} = a\overrightarrow{AB}$ $(0 < a < 1)$.

$\because$ X lie on AB

$\therefore \begin{cases} \overrightarrow{OX} = \overrightarrow{OB} + \overrightarrow{BX} = \overrightarrow{OB} + (1-\lambda)(\overrightarrow{OB} - \overrightarrow{OA}) \\ \overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AX} = \overrightarrow{OA} + \lambda(\overrightarrow{OB} - \overrightarrow{OA}) \end{cases}$

$\therefore \overrightarrow{OX} = a\overrightarrow{OA} + (1-a)\overrightarrow{OB}$

② inverse.

$\overrightarrow{OX} = a(\overrightarrow{OA} - \overrightarrow{OB}) + \overrightarrow{OB}$

$= a\overrightarrow{BA} + \overrightarrow{OB}$

$\therefore$ X's in AB.