HW3:
ODE Exercise

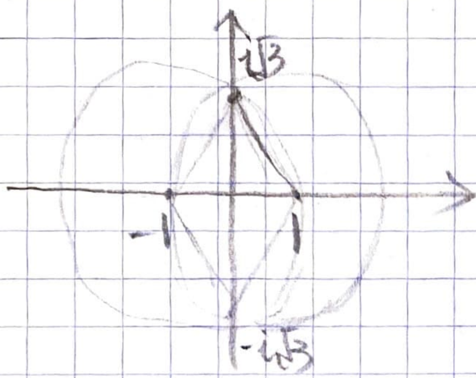
44. Yes. $1, i, 1+i$

45. (a) $3+3i-2i+2=5+i$ (b) $(e^{i\frac{\pi}{3}})^{-1} = e^{-i\frac{\pi}{3}}$

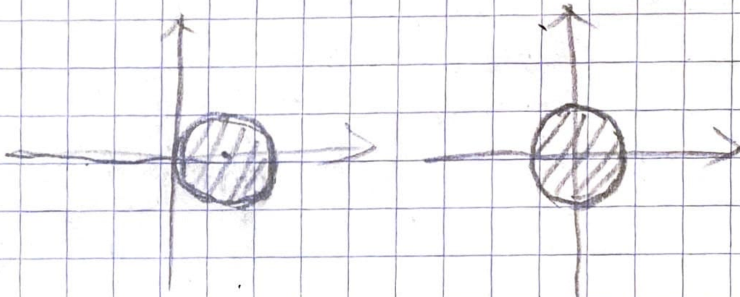
47. $z=a+bi$ $z^{-1}=\frac{1}{a+bi}$ $\bar{z}=a-bi$

$\frac{\bar{z}}{z^{-1}} = (a-bi)(a+bi) = a^2+b^2$

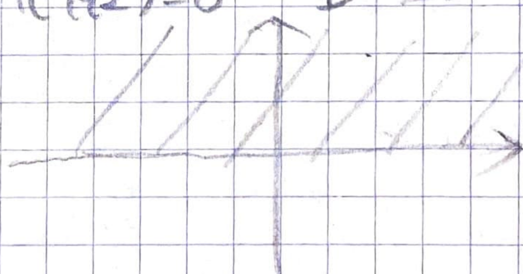
48



49. $|z-1|\le 1$ $|z|\le 1$



$Re(iz)\le 0$ $-b\le 0 \Rightarrow b\ge 0$



51. $\left(\frac{\sqrt{3}}{2}+i\frac{1}{2}\right)^{100}$

$= \left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)^{100}$

$= e^{i\frac{\pi}{6}\cdot 100} = e^{i\frac{50\pi}{3}} = \cos\frac{50\pi}{3}+i\sin\frac{50\pi}{3} = \cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3} = \frac{i\sqrt{3}-1}{2}$

54. $\frac{P(z)}{z-z_0} = q(z)(z-z_0)+P(z_0)$

P is divisible by $z-z_0$ iff $P(z_0)=0$ (i.e. z_0 is a root)

57. Every degree n polynomial can be factored into

$(z-z_1)(z-z_2)(z-z_3)\cdots(z-z_n)$ where z_i is the root

If z_i is in the form $(a+bi)$, there is a z_k in the form $a-bi$ and the product of z_i, z_k is a degree 2 polynomial with real coefficients

58. Vieta's Formula sum of roots: $-\frac{0}{1} = 0$