Linear Algebra

Let $\mathcal{L} = \mathcal{L}(\mathbb{R}^n, \mathbb{R}^m)$, T_A denote the transformation represented by the matrix A

$T_A \circ T_B = T_{AB}$

A norm on V is a function $|\ |: V \to \mathbb{R}$ s.t

$|v| \geq 0$ $\quad |v| = 0 \Leftrightarrow v = 0$ $\quad |\lambda v| = |\lambda||v|$ $\quad |v + w| \leq |v| + |w|$

If V, W are normed spaces, the operator norm of $T: V \to W$ is
$$\|T\| = \sup\left\{\frac{|Tv|_w}{|v|_w} \;\middle|\; v \neq 0\right\}$$

$\|T\|$ is the maximum stretch of T

$\|T \circ S\| \leq \|T\| \, \|S\|$

Let $T: V \to W$. The following are equivalent

a) $\|T\| < \infty$

b) T is uniformly continuous

c) T is continuous

d) T is continuous at the origin

Every linear transformation $T: \mathbb{R}^n \to W$ is continuous

Every isomorphism $T: \mathbb{R}^n \to W$ is a homeomorphism