25) a) f is measurable iff $\mathcal{U}f$ is measurable

$\mathcal{U}f$ is measurable iff $\hat{\mathcal{U}}f$ is measurable

and ~~the~~ $m(\mathcal{U}f) = m(\hat{\mathcal{U}}f)$

$m(\{(x,y) \mid y = f(x)\}) = m(\hat{\mathcal{U}}f \setminus \mathcal{U}f) =$

$m(\hat{\mathcal{U}}f) - m(\mathcal{U}f) = 0$

$\therefore \{(x,y) \mid y = f(x)\}$ is a zero set.

b) Let $A \subset \mathbb{R}$ be a non measurable set

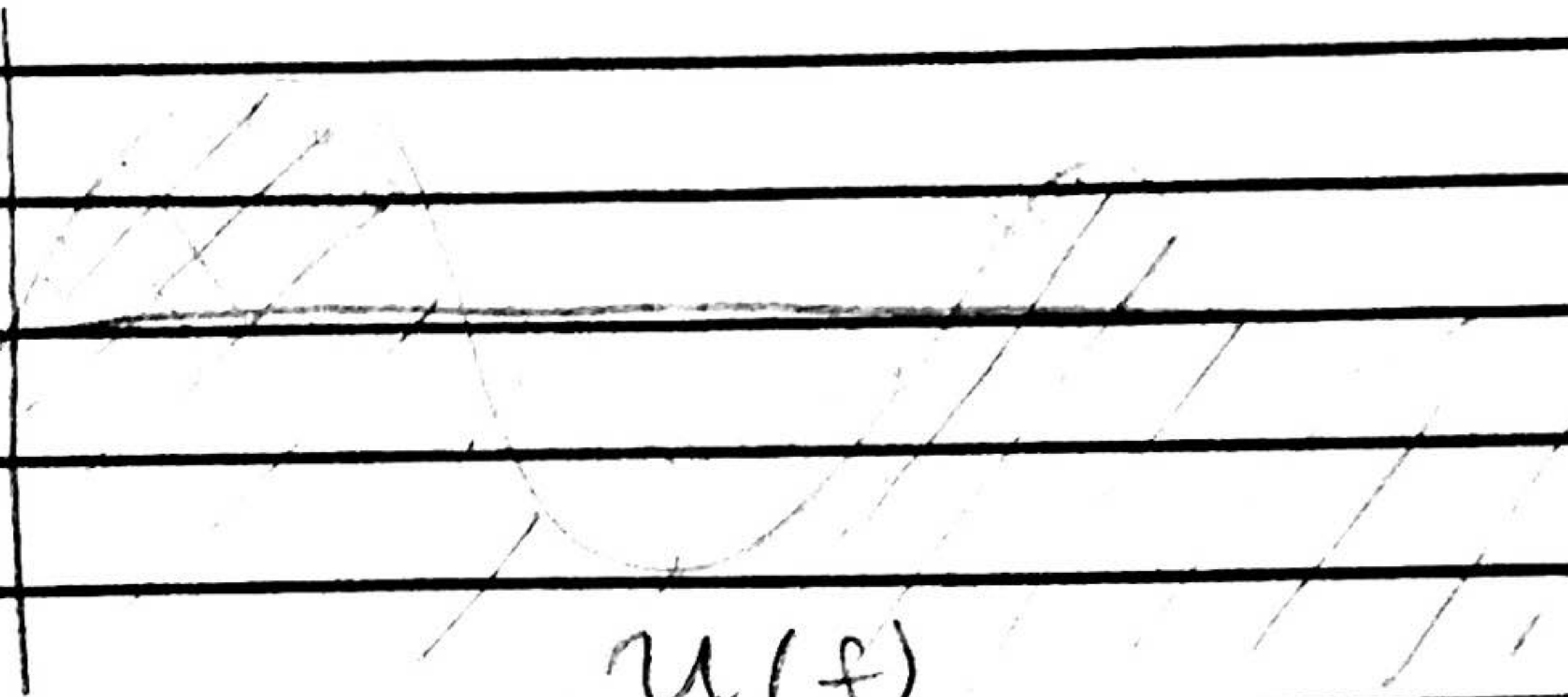
Let $f : \mathbb{R} \to \mathbb{R}$ s.t $f(x) = \begin{cases} 0 & x \in A \\ 1 & x \notin A \end{cases}$

If $E = \{f(x) \mid x \in \mathbb{R}\}$ then $m(E) = 0$ since

any cover of $\{0, 1\}$ will cover E.

However f is a non measurable function

28) a)



$\underline{\mathcal{U}}(f)$

Let $\mathcal{U}f^{+} = \{(x,y) \subset \mathbb{R} \times [0,\infty) \mid y < f(x)\}$

Let $\mathcal{U}f^{-} = \{(x,y) \subset \mathbb{R} \times (-\infty, 0] \mid f(x) < y\}$

$\underline{\mathcal{U}}f = \mathcal{U}f^{+} \cup \mathbb{R} \times (-\infty, 0] \setminus \mathcal{U}f^{-}$

$\mathcal{U}f^{-} \subset \mathbb{R} \times (-\infty, 0]$ is measurable iff

$\mathcal{U}f^{-c} = \mathbb{R} \times (-\infty, 0] \setminus \mathcal{U}f^{-1}$ is measurable

$\mathcal{U}f^{+} \cup \mathbb{R} \times (-\infty, 0] \setminus \mathcal{U}f^{-1}$ is measurable iff

$\mathcal{U}f^{+}$ and $\mathbb{R} \times (-\infty, 0] \setminus \mathcal{U}f^{-1}$ is measurable

$\therefore \mathcal{U}f$ is measurable iff $\mathcal{U}f^{+}$ and $\mathcal{U}f^{-1}$ are measurable

b) Let $f : \mathbb{R} \to (0, \infty)$ be measurable

$\mathcal{U}f = \{(x,y) \mid 0 \le y < f(x)\}$

$T(\mathcal{U}f) = \{(x,y) \mid y > \frac{1}{f(x)}\}$

$(\hat{\mathcal{U}}(\frac{1}{f}))^{c} = \{(x,y) \mid y \le \frac{1}{f(x)}\}^{c} = \{(x,y) \mid y > \frac{1}{f(x)}\}$

$= T(\mathcal{U}f)$

T preserves measurability

$\therefore \mathcal{U}f$ is measurable iff $T(\mathcal{U}f) = (\mathcal{U}(\frac{1}{f}))^{c}$ is measurable

$\mathcal{U}(\frac{1}{f})^{c}$ is measurable iff $\mathcal{U}(\frac{1}{f})$ is measurable

$\therefore f$ is measurable $\Rightarrow \frac{1}{f}$ is measurable.